2  Fundamentals

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2.1 Introduction

Our purpose in this chapter is to present an overview of extrusion machines and pumping mechanisms. We first describe the basic mechanisms of pumping that have been applied to viscous fluids, including both screw and non-screw machines. We then turn to the basic characteristics of screws. Subsequently, we give an overview of the unique characteristics of single screw and the various types of twin screw and multiple screw machines.

The general broad perspective of this chapter has been given in some part in earlier treatises of extrusion, notably the volumes of Schenkel [1], Rauwendaal [2] and Knappe and Potente [3].

2.2 Pumping Mechanisms

2.2.1 General

Many mechanical devices have been invented to pump liquids. These include a wide range of mechanical designs. Some of these devices, such as centrifugal pumps, are suitable only for gases and low-viscosity liquids. Highly viscous liquids are usually
pumped by two different mechanisms: (1) positive displacement pumps, in which fluid enters enclosed chambers and is moved forward by the mechanical movement of the solid parts of the machine; and (2) drag flow pumps, in which the fluid enters a region between two surfaces, one of which is in motion. The relative movement of the two surfaces drags the fluid along a channel, gradually pressurizing it and forced it through a die. Other types of pumping machines also exist for viscous fluids. In this section, we describe these various types of machines.

2.2.2 Positive Displacement Pumps

2.2.2.1 Ram Extruders

Positive displacement pumps include a wide range of mechanical machines. The simplest of positive displacement pumps is the piston pump, which consists of a cylinder containing a liquid to be pumped. Into the top of this cylinder is placed a piston, which is moved along the cylinder axis and pressurizes the liquid. The pressurized liquid is pushed by the piston through an orifice or die placed at its base. This type of machine was the first extruder used in the polymer industry, being applied by R.A. Brooman (4) in 1845 to extrude gutta percha filament (Fig. 2.1). Machines of this type are generally called ram extruders in the polymer industry.

The piston pump or ram extruder has the disadvantage that it is a batch operation. After the liquid in the cylinder has been extruded through the die, it is necessary to remove the piston and add more liquid (or material to be liquefied). There have been efforts to develop continuous flow ram extruders for thermoplastics. Most notable are the inventions by Westover [5] (Fig. 2.2) and Yi and Fenner [6] (Fig. 2.3).

Figure 2.1 Positive displacement piston pump Brooman’s 1845 ram extruder

In the Westover machine, which is described in a 1963 paper [5], four plungers interact with a shuttle valve system. Two feed hoppers on opposite ends of the machine connect to two plasticating ram cylinder systems. These plasticating ram cylinder systems melt the polymer and push it past valves into the die head system. Here two additional
rams push the polymer melt out through the dies. In the Yi-Fenner machine [6], two reciprocating rams feed a barrel containing a rotating shaft that plasticates and melts the thermoplastic.

2.2.2 Rotary Positive Displacement Pumps

More practical positive displacement pumps for viscous liquids involve machines with pairs of intermeshing, counter-rotating rotors, such as those shown in Fig. 2.4 [7–9]. Liquid fed into these machines moves into chambers between the two rotors and the pump walls. These pumps have throughputs given by

\[ Q = 2\pi N V_c \]

(2.1)
where \( Q \) is the volumetric throughput, \( N \) is the rotor rotation rate (rpm), and \( V_c \) is the volume of the filled chamber that moves from the entrance to the exit. These equations represent the behavior of various types of lobe pumps as well as gear pumps. Schmidt’s [8] cam pump (Fig. 2.4a) has two open volumes: one in contact with the inlet and the other in contact with the outlet. His second design (Fig. 2.4b) has two similar volumes plus a third enclosed volume. Gebr. Pintsch’s pump [7] has four lobes and three to four enclosed volumes.

Perhaps the most important of the two intermeshing, counter-rotating rotor positive displacement pumps is the gear pump (Fig. 2.5). Gear pumps are also discussed in the patent literature [10]. The output of a gear pump is given by

\[
Q = 2N V_c = 2N n V_t
\]

where \( V_c \) is the volume of empty chambers around the circumference of a gear, \( n \) is the number such chambers, and \( V_t \) is the individual volume between gear teeth.

Another class of intermeshing counter-rotating, rotor positive displacement pumps is fully intermeshing counter-rotating twin screw pumps [11–16]. Several different designs are shown in Fig. 2.6. The output of such a machine is again

\[
Q = 2N v_i
\]
2.2 Pumping Mechanisms

Figure 2.4b Intermeshing counter-rotating twin rotor pumps: Schmidt design

Figure 2.4c Intermeshing counter-rotating twin rotor pumps: Gebrüder Pintsch design

Figure 2.5 Gear pump
Here \( i \) is the number of thread starts and \( V_c \) is the volume of a C-chamber. We return to fully intermeshing, counter-rotating twin screw machines in Section 2.5.2.

Fully intermeshed, counter-rotating multiscrew pumps are also positive displacement pumps. Examples of these are shown in Fig. 2.7. The output of such a machine is

\[
Q = N(\sum_j V'_{cj})
\]

(2.4)

where \( i \) is the number thread starts and \( V'_{cj} \) the volume of the C-chamber of the screw \( j \).
None of the above machines is a perfect positive displacement pump. There must always be clearances between moving parts and the pressurized viscous fluids being pumped will leak back through them.

### 2.2.3 Drag Flow Pumps

There are a wide range of drag flow machines that can pump viscous fluids. The simplest machine of this type is the drum flow pump or drum extruder shown in Fig. 2.8 [17, 18]. This very fundamental drag flow machine seems to have been invented by Gabrielli [17] in 1952. A similar extruder is described in a later patent by Beck, Robbins, and Birdsall [18]. Here the material to be pumped is introduced into an annular space between the rotating drum and a surrounding barrel. The rotation of the drum carries the liquid to a position where there is a wiper bar and the entrance to a die from which the liquid exits the machines. The wiper bar diverts the liquid into the die. The die pressurizes the liquid and a pressure gradient develops along the length of the channel between the drum and the barrel.

It is readily possible to present a simple flow model of this machine. If the drum has a linear velocity $U$, and there is a uniform clearance $H$ between the drum and the barrel, which has length $W$, the ideal output of the drag flow pump is

$$Q = (1/2)HWU$$

(2.5)
Figure 2.8  (a) Gabrielli’s drum extruder, (b) Beck, Robbins and Birdsall’s drum extruder
The occurrence of a die at the end of the drag flow pump induces backflows along the annular space, which for a Newtonian fluid are proportional to the pressure and inversely proportional to the shear viscosity:

\[
Q = \left(\frac{1}{2}\right) HWU - Q_{\text{back}} \tag{2.6a}
\]

\[
Q = \left(\frac{1}{2}\right) HWU - \frac{KA\Delta p}{\eta} \tag{2.6b}
\]

The single screw extruder is a similar drag flow pump in which the fluid being pumped is dragged along the screw helix. Eq. 2.6 again represents the flow, but it is directed along the helix and not around a drum. The fluid is dragged against the die entrance and pressurization occurs. This pressurization leads to backflow along the screw channel, as in the drum flow pump. The single screw pump is of such importance that we devote Section 2.3 to the geometry of screws and Section 2.4 to screw pump principles and design features.

Drag flow extruders have been developed in which a screw spiral is cut into the surface of a disk [19, 20] (Fig. 2.9). The earliest such machine comes from an anonymous German patent application in March 1944 [19], which did not issue until 1954. A second patent was applied for by Keune in 1949 [20] (when the German patent office was reopened after World War II) and issued in 1951. Since both patents emphasize wire coating, Keune might also be the author of the 1944 application.

Figure 2.9 Screw spiral disk extruder of Keune
Other types of drag flow pumps based on rotating disks have been proposed. One type of machine is described in a 1962 paper by Westover [21]. This is the slider pad or stepped disk extruder (Fig. 2.10). This machine contains a stepped disk positioned a small distance from a flat disk. The flat disk is rotated and as it turns melt is moved into a section of smaller clearance where it is pressurized. When the pressurized melt traverses into the next depressed region it flows through slots out of the extruder.

Still another type of drag flow pump and processing machine, the diskpack, was developed by Tadmor et al. [22, 23] in the late 1970s (Fig. 2.11). A series of stationary, interconnected, hollow disks are placed on smooth rotating shaft. If we introduce a coordinate system into the surface of the rotating shaft, it will be seen that each hollow disk has three dragging surfaces, not one as in a drum extruder, a single screw extruder, or the disk extruders of the previous paragraphs. The shapes of the disks can be modified to allow for solid conveying, melting, melt conveying, mixing, or devolatilization. The diskpack has been commercially developed by Farrel Corporation.

![Figure 2.10 Westover's rotary slider pad pump](image-url)
2.2 Pumping Mechanisms

Maxwell and Scalora [24] have proposed that the normal stress effect discovered by Weissenberg [25] for viscoelastic fluids can be used as the basis of a pump. We show in Fig. 2.12a the normal stress effect as described by Weissenberg [25] and in Fig. 2.12b Maxwell and Scalora’s normal stress based pump. In this machine a viscoelastic fluid (or thermoplastic pellets to be melted) are added at the outer edge of a rotating disk. The fluid when sheared develops both shearing stresses and normal stresses along and perpendicular to the streamlines. The normal stresses act as tensions along the streamlines. The normal stresses along the streamlines may be shown to have through a force balance a radial inward component. This drives the viscoelastic fluid as shown in Figure 2.12a up the stirring rod and in Figure 2.12b radially inward and into the die. The normal stress pump has been commercialized by Custom Scientific as the Mini-Max molder, a laboratory device for molding small quantities of polymers.

Figure 2.11   Tadmor Diskpack extruder

2.2.4   Normal Stress Pumps
The major method of pumping highly viscous fluids involves using screws rotating in barrels. The geometry of a screw located within a barrel is shown in Fig. 2.13. The internal barrel diameter is $D_B$. The diameter of the root of the screw is $D_S$ and the distance between the root of the screw and the internal surface of the barrel is taken as $H$. The screw has a helical flight running along its length. The radial clearance between the crest of the screw flight and the barrel is $\delta_F$. The value of $H$ may vary between the flights, due to a curvature of the screw root. It follows that

$$D_B = D_S(z) + 2H(z)$$  \hspace{1cm} (2.7a)

At the positions of the screw flights,

$$D_B = D_S + 2\delta_F$$  \hspace{1cm} (2.7b)
The axial distance associated with one full turn of the screw flight is the screw lead or pitch, which is defined as $S$. The axial distance between the screw flights is $B$ and the perpendicular distance between the flights is $W$. The flight thickness is designated $e$.

The angle of the screw helix is $\phi$. The screw pitch is related to the helix angle through

$$S = 2\pi \tan \phi$$ \hspace{1cm} (2.8)

Since the screw pitch is independent of radius, the $\phi$ angle must be radius dependent, $\phi(r)$. Thus, at the screw root

$$S = \pi D_S \tan \phi_S$$ \hspace{1cm} (2.9a)

and at the barrel

$$S = \pi D_B \tan \phi_B$$ \hspace{1cm} (2.9b)

Angle $\phi$ decreases as one proceeds from the screw root to the screw barrel.

Many geometric relationships exist between the quantities defined above. The pitch $S$ is related to $B$ and $e$ through

$$S = B + \frac{e}{\cos \phi}$$ \hspace{1cm} (2.10)

The channel width $W$ is related to $B$ through

$$W = B \cos \phi$$ \hspace{1cm} (2.11)

and to the pitch $S$ through substituting Eq. 2.11 into Eq. 2.10:

$$S = \frac{W}{\cos \phi} + \frac{e}{\cos \phi}$$ \hspace{1cm} (2.12a)

and

$$W = \left( S - \frac{e}{\cos \phi} \right) \cos \phi$$ \hspace{1cm} (2.12b)
The relationships given above are for screws with a single lead (Fig. 2.13). It is possible for screws to possess multiple leads or multiple thread starts. If a screw has multiple leads, fluid added to it will flow forward along multiple parallel channels. A screw with a double lead (double flighted) is shown in Fig. 2.14a. A screw with three leads (triple flighted) is shown in Fig. 2.14b. The relationships developed in the first part of this section need to be modified for screws with multiple leads. It should be clear from Fig. 2.14 that for a screw with \( m \) leads

\[
S = m \left( B + \frac{e}{\cos \theta} \right) \quad (2.13a)
\]

\[
W = \frac{S}{m} \cos \phi - e \quad (2.13b)
\]

Figure 2.14  (a) Geometry of a screw with two thread starts; (b) geometry of a screw with three thread starts

Screws have mirror images that are different from each other as one's left hand differs from his right hand (Fig. 2.15). If a rotating right-hand screw drags fluid toward a screw pump exit, a similarly rotating left-handed screw will drag fluid toward the pump entrance (see Section 2.4.1)

Figure 2.15  Right-handed and left-handed screws
2.4 Single Screw Pumps

2.4.1 Principles

The single screw pump is, as we have noted, a drag flow pump and basically operates on the same principles as described in Section 2.2.2.2. The liquid between the screw flights adheres to both the barrel and the screw. The rotation of the screw relative to the barrel drags the liquid along the channel formed by the screw flights. By tradition, a right-handed screw drags liquid from the entrance of the pump to its exit and a left-handed screw drags liquid toward the entrance.

The flow rate of liquid along the screw channel may be expressed analytically in terms of the screw geometry described in Section 2.3. First, we think in terms of a flattened-out screw and barrel and erect a coordinate system $123$ in the root of the channel of the rotating screw in which $1$ is along the channel defined by the flights, $2$ is along the radius of the screw shaft, and $3$ is transverse to the screw flights (Fig. 2.16). In this coordinate system the screw is stationary and the barrel moves with velocity

$$U = U_1 e_1 + U_3 e_3$$

(2.14)

where the velocity $U_1$ along the screw channel is positive and the transverse velocity $U_3$ is negative. $U_1$ and $U_3$ may be written in terms of the screw rotation rate and screw diameter as

$$U_1 = \pi DN \cos \phi \quad U_3 = -\pi DN \sin \phi$$

(2.15)

where $\phi$ is the helix angle of the screw.

Figure 2.16 Coordinate system in screws channel

To a first approximation the flow through a screw channel is drag flow with a linear velocity profile. If we neglect the drag of the screw root and flights, it follows that for a screw with a single lead

$$Q = \left(\frac{1}{2}\right) HW U_1 = \left(\frac{1}{2}\right) \pi HWDN \cos \phi$$

(2.16a)

and if there are $m$ thread starts

$$Q = m \left(\frac{1}{2}\right) \pi HWDN \cos \phi$$

(2.16b)
The discussion above neglects the occurrence of backflows induced by pressure gradients in the screw channel. It is possible, however, to describe these backflows and to devise full velocity profiles in the screw radius or 2 direction through setting up force balances involving flow in the 1 and 3 directions. A shear plane across the screw channel at position \( x_2 \) is introduced. The shear stress on the barrel is \( \sigma_b \) with components \( \sigma_{1b} \) in the 1 direction and \( \sigma_{3b} \) in the 3 direction. The shear stress on the screw flight is \( \sigma_{1s} \) in the 1 direction. These shear stresses are balanced by pressure gradients. The 1 and 3 force balances have the form

\[
W (H - x_2) \frac{dp}{dx_2} = W \sigma_{b1} dx_1 + W \sigma_{12} (x_2) dx_1 + 2 \int_{x_2}^{H} \sigma_{f1} dx' dx_2 \quad (2.17a)
\]

\[
L (H - x_2) \frac{dp}{dx_2} = L \sigma_{b3} dx_3 + L \sigma_{32} (x_2) dx_3 \quad (2.17b)
\]

where differential slices of thicknesses \( dx_1 \) and \( dx_3 \) are used and \( \sigma_{12} \) and \( \sigma_{32} \) are shear stresses at positions \( x_2 \) in the screw channel in the 1 and 3 directions. \( L \) is a length along the screw flights. If friction on the screw flight walls is neglected, Eqs. 2.17a and 2.17b are equivalent to

\[
\sigma_{12} (x_2) = (H - x_2) \frac{\partial p}{\partial x_1} - \sigma_{b1} \quad (2.18a)
\]

\[
\sigma_{32} (x_2) = (H - x_2) \frac{\partial p}{\partial x_3} - \sigma_{b3} \quad (2.18b)
\]

If we presume that the fluid being pumped is Newtonian, the shear stress is proportional to the shear rate and we may write

\[
\sigma_{12} = \eta \frac{\partial v_1}{\partial x_2} \quad (2.19a)
\]

\[
\sigma_{32} = \eta \frac{\partial v_3}{\partial x_2} \quad (2.19b)
\]

Substitution of Eqs. 2.19a and 2.19b into Eqs. 2.18a and 2.18b leads to ordinary differential equations for the velocity components \( v_1 \) and \( v_3 \) as a function of \( x_2 \). These equations may be solved using Eq. (2.15) as boundary conditions, which gives

\[
v_1(x_2) = U_1 \frac{x_2}{h} - \frac{H^2}{2\eta} \frac{\partial p}{\partial x_1} \left[ \frac{x_2}{H} - \left( \frac{x_2}{H} \right)^2 \right] \quad (2.20a)
\]

\[
v_3(x_2) = U_3 \frac{x_2}{h} - \frac{H^2}{2\eta} \frac{\partial p}{\partial x_3} \left[ \frac{x_2}{H} - \left( \frac{x_2}{H} \right)^2 \right] \quad (2.20b)
\]
In arriving at Eq. 2.20, we have treated $\sigma_{b1}$ and $\sigma_{b3}$ as unknown constants and eliminated them to satisfy boundary conditions. This is shown in Fig. 2.17a.

The flow rate along the screw channels is

$$Q = W \int_0^H v_1 dx_2 = \frac{U_0 H W}{2} - \frac{H^3}{12 \eta} \frac{\partial p}{\partial x_1} = \pi D H W N \cos \phi - \frac{H^3}{12 \eta} \frac{\partial p}{\partial x_1}$$

(2.21)

The drag of the barrel due to the rotation of the screw moves the polymer melt forward along the screw channel between the flights.

The velocity component perpendicular to the screw flights, $v_3(x_2)$, is also of interest. Note that

$$\int_0^H v_3 dx_2 = \frac{U_0 H}{2} - \frac{H^3}{12 \eta} \frac{\partial p}{\partial x_3} = 0$$

(2.22a)

where

$$U_3 = -\pi D N \sin \phi$$

(2.22b)

There is a pressure gradient across the screw channel given by

$$\frac{\partial p}{\partial x_3} = -\frac{12}{H^2} \pi D N \sin \phi$$

(2.23)

The pressure is a maximum at the leading screw flight and then decreases in the normal direction away from the flight in a linear fashion. This results in a velocity field of form

$$v_3(x_2) = -\pi D N \sin \phi \left[ 3 \left( \frac{x_2}{H} \right)^2 - 2 \frac{x_2}{H} \right]$$

(2.24)

At large $x_2/H$, the fluid is dragged in a negative direction along the barrel. At small $x_2/H$, there is a positive pressure. The result is a circulating flow, shown in Fig. 2.17b.

The velocity field in the down-channel and cross-channel directions are shown in Fig. 2.17. The down-channel flow is dominated by forward drag. The transverse motion is circulatory. The melt flows in a helical manner in the screw channel. This is, of course, superposed on the flow screw helix.

The primary results of this section, Eqs. 2.20 to 2.24, are well known and were developed in the literature by various authors in the period 1925 to 1959 [2, 3].
2.4.2 Simple Screw Design Features

It is possible to interpret the influence of screw design variables such as channel depth, width, and helix angle on fluid/melt characteristics such as pressurization using Eq. 2.21, which we may write as

\[
Q = \frac{1}{2} \pi D^3 \eta H \cos \phi \sin \phi - \frac{\pi H D}{12 \eta} \frac{\partial p}{\partial z} \sin^2 \phi
\]

where we have written \( W = \pi D \sin \phi \) and \( x_1 = z / \sin \phi \), where \( z \) is the screw axis direction.

Let us analyze Eq. 2.25. First, a large viscosity \( \eta \) reduces backflow and makes the pump more efficient. We next consider the influence of screw channel depth \( H \). Clearly, if \( H \) is decreased the backflow leakage must fall off more rapidly than the forward drag flow. Thus, small \( H \) makes a more effective pump.

In the absence of pressure flow, the maximum output is obtained at \( \phi = 45^\circ \). This follows from \( \cos \phi \sin \phi \) being \( (1/2) \sin 2\phi \). Including pressure flow reduces the value of the optimum \( \phi \).
2.5 Counter-Rotating Twin Screw Machine

2.5.1 Tangential

Tangential counter-rotating (Fig. 2.18) twin screw extruders as envisaged in the work of Fuller [26] are essentially drag flow pumps with the drag being supplied by the fraction of the screw circumference that is in contact with the barrel. These screw machines can have matched screw flight or staggered screw flight configurations (Fig. 2.18).

![Figure 2.18 Tangential counter-rotating twin screw pump: (a) matched flights; (b) staggered flights](image)

The concept of this flow mechanism was developed by Kaplan and Tadmor [27], whom we follow here. Essentially between the screw and the barrel, we may write, as in a single screw extruder,

$$Q = \frac{U_1 HW}{2} \frac{H^3W}{12\eta L_I} \frac{\Delta p_I}{L_I}$$  \hspace{1cm} (2.26)

where $\Delta p_I$ and $L_I$ refer to pressure drop and channel length in contrast with the barrel. Between the screws (region II) we have only pressure flow for matched screw flights

$$Q = -\frac{H^3W}{3\eta} \frac{\Delta p_{II}}{L_{II}}$$  \hspace{1cm} (2.27)

where we have considered that in this region the channel dimensions $2H$ by $W$. The total pressure drop per revolution is

$$\Delta p = \Delta p_I + \Delta p_{II}$$  \hspace{1cm} (2.28)

These machines can have matched screw flight or staggered screw flight configurations (Fig. 2.18). Eq. 2.28 leads to
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\[ Q = \frac{1}{2} WHU \left( \frac{4f}{3f+1} \right) - \frac{WH^3}{12\eta} \frac{\Delta p}{L_1 + L_{II}} \left( \frac{4}{3f+1} \right) \]  \hspace{1cm} (2.29)

where

\[ f = \frac{L_1}{L_1 + L_{II}} \]  \hspace{1cm} (2.30)

Since the quantity \( f \) is less than unity, drag flow is reduced and backward pressure flow for each screw is increased.

The total flow \( Q_T \) for the tangential twin screw extruder encompasses expressions such as Eq. 2.29 for each screw and a backward interscrew leakage, specifically

\[ Q_T = 2Q - Q_{\text{leak}} \]  \hspace{1cm} (2.31)

or, using Eq. 2.14,

\[ Q_T = WH\pi DN \cos \phi \left( \frac{4f}{3f+1} \right) - \frac{WH^3}{6\eta} \frac{\Delta p}{L_1 + L_{II}} \left( \frac{4}{3f+1} \right) - Q_{\text{leak}} \]  \hspace{1cm} (2.32)

Here \( Q_{\text{leak}} \) is the backward flow through the triangular region between the screws. Kaplan and Tadmor estimate \( Q_{\text{leak}} \) as the backward pressure flow through a uniform triangular section:

\[ Q_{\text{leak}} = \frac{W_T H_T S}{12\eta} \left( \frac{\Delta p}{\Delta L} \right) \beta \]  \hspace{1cm} (2.33)

where \( W_T \) is the base of the triangle, \( H_T \) is the height of the triangle, \( \Delta p/\Delta L \) is the pressure gradient flight to flight along the screw axis, and \( S \) is a shade factor.

The derivation cited above is valid only for matched screw flights. If the screw flights on the two screws are staggered relative to each other, the behavior will change. Most strikingly, the triangular cross-section through which backward leakage increases in size and the pumping characteristics become worse. Analytical models for flow in staggered flight geometry are given by Rauwendaal \[2\] and included in the study of White and Adewale \[28\]. There have also been a number of numerical simulations. This subject is developed in White \[29\] and in Section 6.4.

### 2.5.2 Intermeshed

As described in Section 2.3.2, the fully intermeshing, counter-rotating twin screw extruder is a positive displacement pump. The machine consists of alternating thick flights and screw channels on the two parallel opposite screws. As the screws rotate, the open channels, which have C shapes, may be observed to move forward along the screw axis (Fig. 2.19). The volume of material in these C-chambers is pumped forward by the rotation of the screws to the die.
The output of the machine was discussed by Kiesskalt [30] and Montelius [16] in the
1920s and in various books [1–3, 29, 31] since that time. It is given by Eq. 2.3, which we
repeat here:

\[ Q = 2NiV_c \]  

(2.3)

where \( i \) represents the number of thread starts and \( V_c \) the volume of the C-chamber. The
volume of the C-chamber is to a first approximation:

\[ V_c = \frac{\pi DH W_m}{\cos \phi_m} \]  

(2.34)

where \( W_m \) is the mean channel width and \( \phi_m \) is the mean flight angles.

The fluid mechanics of intermeshing, counter-rotating twin screw extruders has
generally been simulated based on a model where the coordinate system is fixed on the
surface of a screw translating with a C-chamber [1–3, 29–33]. The C-chamber translates
along the screw axis at a velocity

\[ U_n = SN = \pi DN \tan \phi \]  

(2.35)

The equations of motion for a Newtonian fluid may readily solved for a Newtonian fluid
in a C-chamber under conditions where transverse shearing by the walls of the screw
channel is neglected. The 1 and 3 velocity components parallel and transverse to the screw
channel directions are of the same form as Eq. 2.20a,b, except for consideration of the
translating coordinate system. Specifically, these are

\[ v_1(x_2) = -\frac{\pi DN}{\cos \phi} + \frac{\pi DN}{\cos \phi} \cos \phi \left( \frac{x_2}{H} \right) - \frac{H^2}{2\eta} \frac{\partial p}{\partial x_1} \left[ \frac{x_2}{H} - \left( \frac{x_2}{H} \right)^2 \right] \]  

(2.36a)

\[ v_3(x_2) = -\pi DN \sin \phi \left( \frac{x_2}{H} \right) - \frac{H^2}{2\eta} \frac{\partial p}{\partial x_3} \left[ \frac{x_2}{H} - \left( \frac{x_2}{H} \right)^2 \right] \]  

(2.36b)
The total flux in the C-chamber-based coordinate system is zero, so that

\[ \int_0^H v_1 dx_2 = 0 \]  

(2.37a)

\[ 0 = \frac{\pi DN}{\cos \phi} HW + \frac{1}{2} \pi DN \cos \phi HW - \frac{WH^3}{12 \eta} \frac{dp}{dx_1} \]  

(2.37b)

The pressure gradient in the C-chamber is

\[ \frac{dp}{dx_1} = -6\eta x_1 \frac{D^2}{H^2} \left( \frac{2}{\cos \phi} - \cos \phi \right) < 0 \]  

(2.38)

The pressure is the highest at the back of the C-chamber and the lowest in the front.

The pressurized liquid in the C-chamber tends to leak back between the machine clearances. Doboczky [32] and later Janssen et al. [31, 33] and most recently White and Adewale [28] sought to characterize the various leakage flows. Most important is backflow through the calendering gap.

We return to the intermeshing, counter-rotating twin screw extruder in Section 6.3.

### 2.6 Co-Rotating Twin Screw Machine

The co-rotating twin screw extruder used in modern technology is, as explained in Section 1.2, a modular, intermeshing, self-wiping machine. The basic self-wiping, co-rotating screw machine is shown in Fig. 2.20. This figure is taken from a patent by Wunsche [34] who first discussed such a machine. In the self-wiping, co-rotating twin screw extruder, the channels are open. Forward pumping operates on the principle of drag flow. As described by Erdmenger [35], the pioneer of the modern machine, the fluid moves forward along the screw channel in a figure-eight pattern.

As the screw channel traces the barrel along its path, we may write, if we neglect transverse shearing, the forward velocity profile \( v_1(x_2, x_3) \) in the form of Eq. 2.20a:

\[ v_1(x_2, x_3) = U_1 \frac{x_2}{H} - \frac{H^2}{2 \eta} \frac{dp}{dx_1} \left( \frac{x_2}{H} - \frac{x_2^2}{H^2} \right) \]  

(2.39)

Figure 2.20  Self-wiping, co-rotating twin screw pump
2.7 Multiple Screw Extrusion

The flux \( q_1 \) is

\[
q_1 = \int_0^H v_1 \, dx_2 = \frac{1}{2} U_1 H - \frac{H^3}{12 \eta} \frac{\partial p}{\partial x_1}
\]  

(2.40)

The total flow \( Q \) is

\[
Q = \int_0^{w_{\text{max}}} q_1 \, dx_3 = \frac{1}{2} U_1 A - \frac{1}{12 \eta} \left( H^3 \frac{\partial p}{\partial x_1} \right)
\]

(2.41)

The basic mechanism of flow in a co-rotating twin screw extruder was perhaps first discussed by Erdmenger [35] and more especially Herrmann and Burkhardt [36], who presented a simple fluid mechanical analysis of the flow. The co-rotating twin screw machine is reviewed by Rauwendaal [2] and more especially by White [29].

It should be noted that the above analysis is for a uniform fully filled machine and in practice co-rotating twin screw extruders are operated under starved conditions and are modular in construction. This will be addressed in Chapter 6.

We return to this machine in Section 6.2.

2.7 Multiple Screw Extrusion

While twin screw extrusion machines have become important, there has also been considerable activity through the years on multiple screw extrusion machines. Most attention to multiple screw machines has been for intermeshing, counter-rotating twin screw machines (see Fig. 2.21), which are all positive displacement pumps. An explicit discussion of multiple fully intermeshing, counter-rotating screw pumps is contained in a 1925 patent application of Montelius [14] and in a 1933 paper [37] by the same author. These pumps were clearly being manufactured commercially by the 1930s. The output of a multiple fully intermeshing counter-rotating twin screw machine is clearly given by Eq. 1. Fig. 2.21 shows three screw and five screw intermeshing, counter-rotating screw pumps [38, 39].

Self-wiping co-rotating, multiple screw pumps have been described by Colombo [40] and Meskat and Erdmenger [41] among others (see Fig. 2.22). These are drag flow pumps. Multiple screw machines involving both co-rotating and counter-rotating screws have also been proposed. A four screw machine involving two pairs of self-wiping, co-rotating screws that are tangentially counter-rotating with each other was proposed by Erdmenger and Oetke [42] (Fig. 2.23). This machine was used for devolatilization and operates by drag flow.
Figure 2.21 Multiple screw, intermeshing, counter-rotating screw pumps: (a) Three screws; (b) Five screws

Figure 2.22a Self-wiping, co-rotating multiple screw pumps: multiple co-rotating screw extruders after Colombo [40] indicating (i) 3- and 5-screw linear arrangements, (ii) 6- and 8-screw circumferential arrangements, and (iii) 4-screw arrangement with contact 65 spars